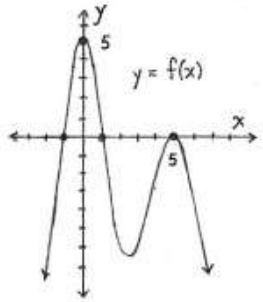
Section 4.4

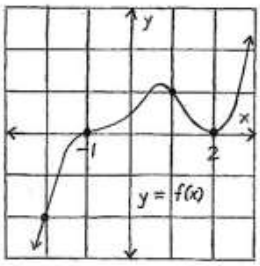
Section 4.4 (cont'd)

In Exercises 66-67, find the CFF of the polynomial (assume min degree).

(66)



(67)



66

According to the graph, the zeros are

-1 and 1 (each with multiplicity one because the graph crosses the x-axis at these points)

5 (with multiplicity two because the graph touches the x-axis at this point)

Therefore,

$f(x) = k(x+1)(x-1)(x-5)^2$ and we must solve for "k" so that f(0) = 5

$f(0) = k(1)(-1)(-5)^2 = 5$

$-25k = 5$

$k = \frac{-1}{5}$

The complete factored form is

$f(x) = \frac{-1}{5}(x+1)(x-1)(x-5)^2$

67

According to the graph, the zeros are

-1 (with multiplicity three because it looks like a cubic function at this point)

2 (with multiplicity two because the graph touches the x-axis at this point)

Therefore,

$f(x) = k(x+1)^3(x-2)^2$ and we must solve for "k" so that f(-2) = -2

$f(-2) = k(-1)^3(-4)^2$

$-2 = -16k$

$k = \frac{1}{8}$

The complete factored form is

$f(x) = \frac{1}{8}(x+1)^3(x-2)^2$

68

(68) State the multiplicity of the zeros of $f(x) = x^3(x-2)(x-5)^4$.

The zeros of the function are

0 (multiplicity 3)

2 (multiplicity 1)

5 (multiplicity 4)

69

(69) Determine where the graph of the polynomial in Exercise 68 crosses or is tangent to the x-axis.

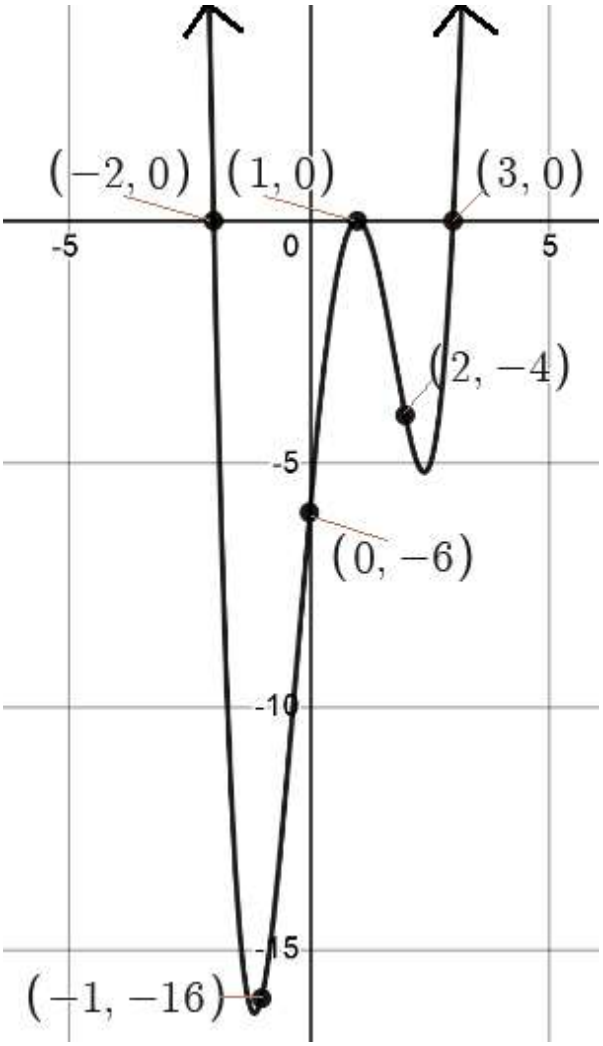
The graph crosses the x-axis at zeros of odd multiplicities and touches the x-axis at zeros of even multiplicities.
The polynomial function in # 68 crosses the x-axis at the points (0, 0) and (2, 0).
On the other hand, the polynomial function is tangent to the x-axis at the point (5, 0).

70

(70) Sketch the graph of the polynomial $f(x) = (x+2)(x-1)^2(x-3)$.

The zeros of the function are -2 (multiplicity one), 1 (multiplicity two), and 3 (multiplicity one).
Therefore, the graph crosses the x-axis at the points (-2, 0) and (3, 0) and the graph is tangent to the x-axis at (1, 0).
The leading coefficient is positive and the degree is 4 which is an even number so the end behavior is that the left side and right side of the graph both rises.
$f(0) = (0+2)(0-1)^2(0-3) = -6$ so the y-intercept is (0, -6)
In addition,
$f(-1) = (-1+2)(-1-1)^2(-1-3) = -16$ so the point (-1, 16) is on the graph

$f(2) = (2+2)(2-1)^2(2-3) = -4$ so the point (2, -4) is on the graph but we have no evidence that this is a turning point.
Below shows the graph of the polynomial function



Section 4.5

Section 4.5

(71) Find the complete factored form of a cubic polynomial with real coefficients, a leading coefficient of -1 and zeros 5, 3i and -3i.

71

We have

$x = 5, x = 3i$, and $x = -3i$
Transpose the constants to the left side of each equation
$x - 5,\ x - 3i = 0,$ and $x + 3i = 0$
Multiply the expressions on the left side
$(x - 5)(x - 3i)(x + 3i) = 0$
$(x - 5)[x^2 - (3i)^2] = 0$
$(x - 5)[x^2 - (-9)] = 0$
$(x - 5)(x^2 + 9) = 0$
Since the leading coefficient is -1, multiply both sides by -1
$-(x - 5)(x^2 + 9) = 0$
Answer: $f(x) = -(x - 5)(x^2 + 9)$
Note: There must be a "negative sign" in front

72

(72) Express your answer in Exercise 71 in expanded form.

We have

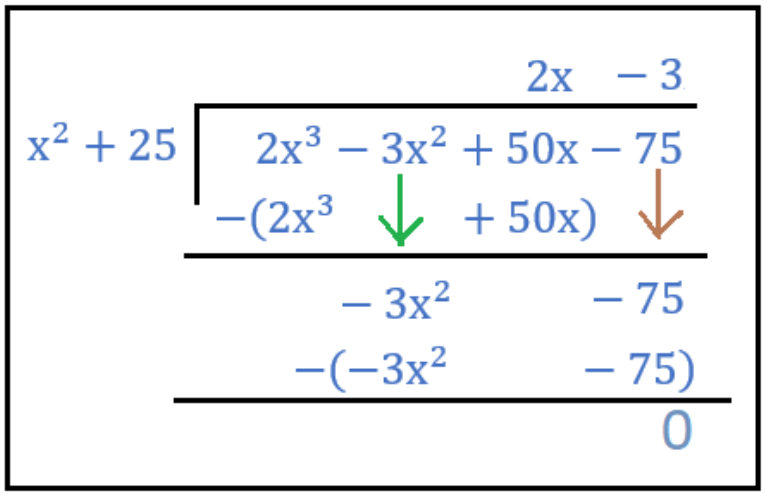
$$-(x - 5)(x^2 + 9) = -(x^3 - 5x^2 + 9x - 45)$$
$$= -x^3 + 5x^2 - 9x + 45$$

Answer: $f(x) = -x^3 + 5x^2 - 9x + 45$

73

(73) Find the zeros of $f(x) = 2x^3 - 3x^2 + 50x - 75$ given $5i$ is a zero.

The conjugate of $5i$ is $-5i$ so this is the other zero
$x = 5i$ or $x = -5i$
$(x - 5i)(x + 5i) = 0$
$x^2 - (5i)^2 = 0$
$x^2 - (-25) = 0$
$x^2 + 25 = 0$
To get the other zero, we divide the polynomial by $x^2 + 25$ using long division



The quotient is $2x - 3$
Hence, we solve $2x - 3 = 0$
$x = 3/2$
The zeros are $5i$, $-5i$, and 3/2

#74

(74) Write the complete factored form of Exercise 73.

By long division in the previous part, we have shown that

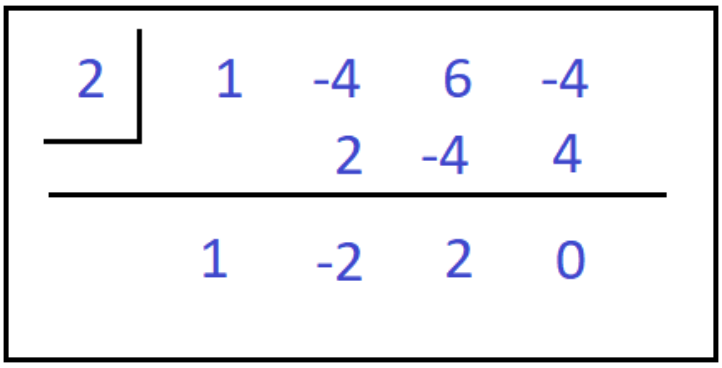
$f(x) = (x^2 + 25)(2x - 3)$

#75

(75) Solve the polynomial equation $x^3 - 4x^2 + 6x - 4 = 0$.

$f(2) = 2^3 - 4(2^2) + 6(2) - 4 = 0$ so $x - 2$ is a factor

Using synthetic division, we have



The quotient is $x^2 - 2x + 2$ so we solve $x^2 - 2x + 2 = 0$

Using quadratic formula,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a = 1, b = -2$, and $c = 2$ (coefficients)

$= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$

$= \frac{2 \pm \sqrt{-4}}{2}$ Note: $\sqrt{-4} = \sqrt{4}\sqrt{-1} = 2i$

$= \frac{2 \pm 2i}{2}$

$= 1 \pm i$

Answer: 2, $1 + i$, and $1 - i$